

Proportionality and Strategyproofness in Multiwinner Elections

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Multiwinner Elections

- Given approval profile, select a committee of exactly k candidates.
- Find good committee *rule*: $f : (2^A)^n \rightarrow \mathcal{C}_k$
- One option (AV): choose k candidates with highest approval score
— strategyproof!
- e.g. $(ab, ab, ab, de, df) \rightarrow$ AV committee ab ; proportional ad

Proportional Approval Voting (PAV)

- Select the k -committee W maximising

$$\sum_{i \in N} \left(1 + \frac{1}{2} + \dots + \frac{1}{|P(i) \cap W|} \right)$$

- Diminishing marginal utility
- Proportional: 30% group gets ~30% of seats
- Vulnerable to free-riding
Profile $(abc, abc, abc, abd, abd) \rightarrow abc$
 $(abc, abc, abc, abd, d) \rightarrow abd$

Is there a rule which is both
proportional and strategyproof?

(No.)

Is there a rule which is both
proportional and strategyproof?

Theorem. Suppose $k \geq 3$ and n is a multiple of k . No resolute rule satisfies proportionality and strategyproofness.

Strategyproofness

Cardinality-Strategyproofness If P' is an i -variant of P , then we do not have $|f(P') \cap P(i)| > |f(P) \cap P(i)|$.

Hamming-Strategyproofness If P' is an i -variant of P , then we do not have $\mathcal{H}(f(P'), P(i)) < \mathcal{H}(f(P), P(i))$.

Superset-Strategyproofness If P' is an i -variant of P , then we do not have $f(P') \cap P(i) \supsetneq f(P) \cap P(i)$.

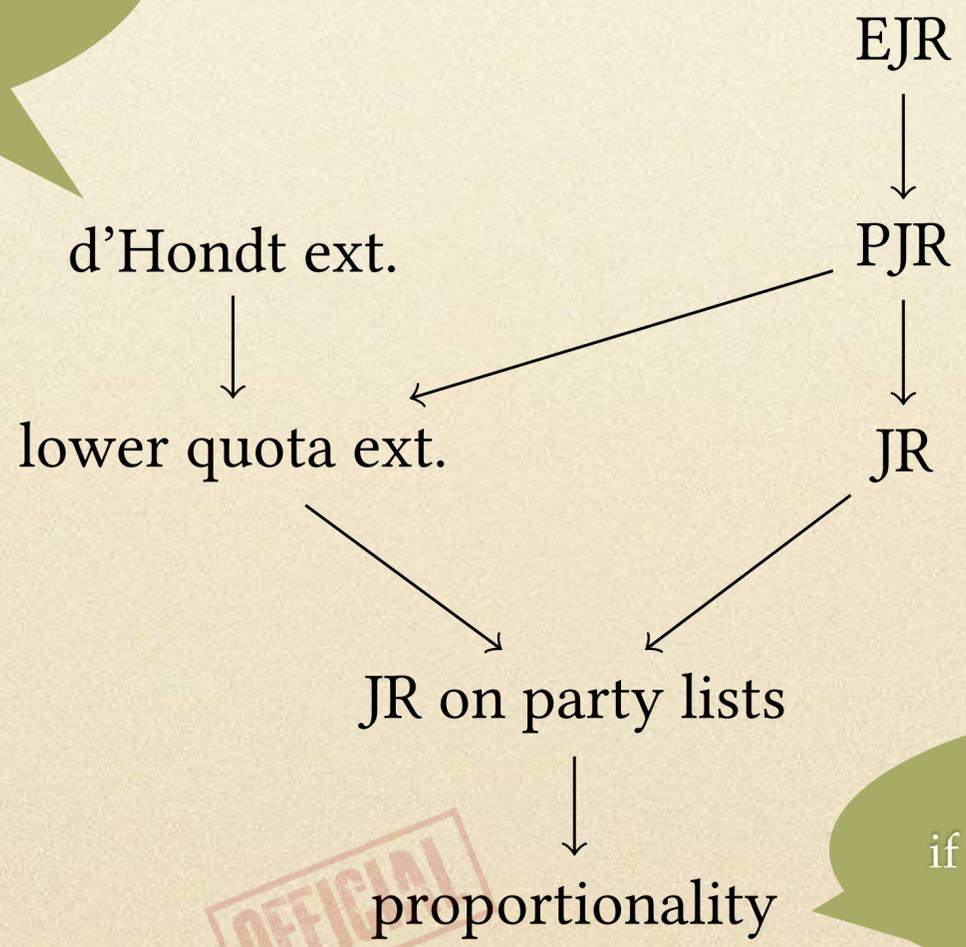
Strategyproofness If P' is an i -variant of P with $P'(i) \subset P(i)$, then we do not have $f(P') \cap P(i) \supsetneq f(P) \cap P(i)$.

OFFICIAL

Proportionality

committee rule induces
apportionment method over
party-list profiles
(*ab,ab,ab,cd,cd*)

cohesive group of
 $l \cdot \frac{n}{k}$ voters deserves
 l representatives



cohesive group of
 n/k voters deserves
a representatives

can also use
disjoint diversity

in party-list profile with $\leq k$
every party represented

in a party-list profile,
if singleton party {a} has support
 n/k , then put *a* in committee



SAT Solving

ALGORITHM 1: Encode Problem for SAT Solving

Input: Set C of candidates, set N of voters, committee size k .

Question: Does a proportional and strategyproof committee rule exist?

for each profile $P \in \mathcal{B}^N$ **do**

if P is a party-list profile **then**

$\text{allowed}[P] \leftarrow \{C \in C_k : C \text{ provides JR to singleton parties}\}$

else

$\text{allowed}[P] \leftarrow C_k$

for each committee $C \in \text{allowed}[P]$ **do**

 introduce propositional variable $x_{P,C}$

for each profile $P \in \mathcal{B}^N$ **do**

 add clause $\bigvee_{C \in \text{allowed}[P]} x_{P,C}$

 add clauses $\bigwedge_{C \neq C' \in \text{allowed}[P]} (\neg x_{P,C} \vee \neg x_{P,C'})$

for each voter $i \in N$ **do**

for each i -variant P' of P with $P'(i) \subseteq P(i)$ **do**

for each $C \in \text{allowed}[P]$ and $C' \in \text{allowed}[P']$ **do**

if $C' \cap P(i) \supsetneq C \cap P(i)$ **then**

 add clause $(\neg x_{P,C} \vee \neg x_{P',C'})$

pass formula to SAT solver

return whether formula is satisfiable

Applications to:

- Tournament solutions
- Voting
- Randomised voting
- Fair budgeting
- House allocation

SAT \rightarrow good rule

UNSAT \rightarrow impossibility

Minimal Unsatisfiable Set

yields human-readable proof

Proof Strategy

- Using SAT + MUS, prove base case $k = 3, n = 3, m = 4$.
 - Consider profile $P = (ab, c, d)$; by proportionality wlog $f(P) = acd$.
 - Use strategyproofness to deduce value of f at 13 other profiles, obtaining contradiction $f(P) = bcd$.
- Prove induction steps:
 - By copying the profile, holds for $q \cdot k$ voters
 - By adding never-approved candidates, holds for $m \geq 3$
 - By adding a $k+1$ st 'party', can move from k to $k + 1$

$k = 2?$

other $n?$

what can we say
without?

Theorem. Suppose $k \geq 3$ and n is a multiple of k . No resolute rule satisfies proportionality and strategyproofness.

Small Committees

- Result does not hold for $k = 2$.
- For $k = 2$ and n odd, AV satisfies JR!
 - because cohesive groups are strict majorities, which AV favours
- But does hold when allowing all manipulations (not just subsets).

Other Electorate Sizes

- Is the assumption that n is a multiple of k necessary?
 - SAT solver suggests yes, at least for small parameters
 - Open question
- Result holds for *all* n when using Droop quota:
every group of $> n/(k + 1)$ needs to be represented.

Resoluteness

- Proof exploits “awkward resoluteness”
 $P = (ab, c, d)$; f needs to make an arbitrary choice
- How to relax resoluteness?
- Result holds for randomised rules:
 - Proportionality: all committees in support must be proportional
 - Strategyproofness: Can't get more approved candidates in expectation

Open Questions

- Does result hold for all n ?
- What about single-peaked preferences?
intuitively, choosing e.g. percentiles is proportional
- Proportionality and (strong) monotonicity
- EJR and committee monotonicity
- Characterise AV with reinforcement + s.p. (cf. Lackner & Skowron IJCAI-18)