Generalizing Instant Runoff Voting to Allow Indifferences

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Introduction

Instant Runoff Voting (IRV) is a widely used single-winner voting rule based on (possibly truncated) linear orders. Many electoral reform advocates prefer this method.

This method repeatedly eliminates the candidate that is ranked top least often, until only one candidate remains who is the winner.

We ask: what is the right way to generalize IRV to weak orders (allowing indifferences)? Later in the talk I explain why this question is interesting.

1	1	1
c	a b	d
a	c	b
b	d	a
d		c

$\rightarrow$

1	0.5	0.5	1
c	a	b	d
a	b	a	b
b	c	c	a
d	d	d	c

Figure 1: Replacing weak orders by weighted linear orders.

General methods for defining voting rules for weak orders: C2 methods (not applicable), replacing weak orders by a mixture of linear orders (in general exponential time, but easy for IRV).

Approval-IRV

We propose Approval-IRV. At each step, interpret each vote as an approval vote for its top-ranked uneliminated alternatives. Delete the candidate with the fewest approvals.

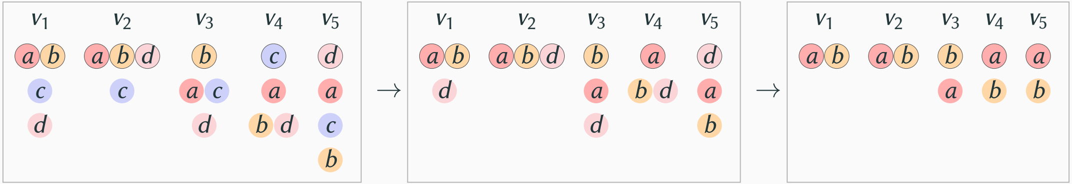


Figure 2: An example of Approval-IRV with voters $v_1, \dots, v_5$. The first eliminated alternative is c, which is ranked on top only once. Then d is eliminated, and finally a wins the majority vote against b. Thus, a is the winner.

Alternative method: Split-IRV where a vote with 3 top-ranked alternatives gives $\frac{1}{3}$ points to these alternatives, and the lowest-scoring alternative is deleted. Equivalent to running IRV after the replacement operation.

History

Multiwinner Split-STV was developed in series of articles in the journal *Voting Matters*.

> Brian L. Meek. "A new approach to the Single Transferable Vote. Paper II: The problem of non-transferable votes". In: *Voting matters* (1 1994). URL: https://www.votingmatters.org.uk/issue1/P2.htm

> C. Hugh E. Warren. "STV and equality of preference". In: *Voting matters* (7 1996). URL: https://www.votingmatters.org.uk/issue7/P5.htm

Split-STV "was first used by the John Muir Trust (for Trustee elections) in 1998, and by the London Mathematical Society in 1999" and both still use Split-STV today

> Denis Mollison. "Fair votes in practice". In: *arXiv:2303.15310* (2023). URL: https://arxiv.org/abs/2303.15310

The only previous scholarly discussion of Approval-STV is by Janson (2016).

> Svante Janson. "Phragmén's and Thiele's election methods". In: *arXiv:1611.08826* (2016). URL: https://arxiv.org/abs/1611.08826

Since about 1996, there have been sporadic discussions of Approval-IRV on internet forums, see e.g., the election-methods mailing list (1996, 2004), electowiki, and reddit (2019). A 2004 webtool implements both Approval-IRV and Split-IRV.

Axiomatic Comparison

	Approval-IRV	Split-IRV
Independence of clones	✓	✗
Respecting cohesive majorities	✓	✗
Indifference monotonicity	✓	✗

Table 1: Comparison of properties satisfied by the rules.

Outline:

- Explain these three axioms and the claims in the table.
- State our characterization theorems.
- Motivation for moving to weak orders.
- Experiments.
- Multi-winner STV.

Independence of Clones

clone sets:	a	x_1	a	not clone sets:	x_1	a
	x_1	x_2	$x_1\ x_2\ x_3\ b$		$a\ x_2$	$b\ x_1$
	$x_2\ x_3$	x_3	c		x_3	$x_2\ x_3$
	$b\ c$	$a\ b\ c$			$b\ c$	c

Figure 3: Examples of $X = \{x_1, x_2, x_3\}$ being a clone set or not being a clone set.

$X \subseteq C$ is a *clone set* if for every $i \in N$ and every candidate $c \notin X$, either

$$x \succ_i c \text{ for all } x \in X, \quad \text{or} \quad x \sim_i c \text{ for all } x \in X, \quad \text{or} \quad c \succ_i x \text{ for all } x \in X.$$

Independence of Clones

Definition

A voting rule f satisfies *independence of clones* if for all profiles P with clone set $X \subseteq C$, letting $\hat{P}$ be the profile obtained by removing all but one candidate $\hat{x}$ from X, we have

1. for every $c \notin X$, we have $c \in f(P)$ if and only if $c \in f(\hat{P})$, and
2. $\hat{x} \in f(\hat{P})$ if and only if there exists $x \in X$ such that $x \in f(P)$.

T. Nicolaus Tideman. "Independence of clones as a criterion for voting rules". In: *Social Choice and Welfare* 4 (1987), pp. 185–206. DOI: 10.1007/BF00433944. URL: https://www.condorcet.vote/view/DOCS/IndependenceofClones.pdf

Markus Schulze. "A new monotonic, clone-independent, reversal symmetric, and condorcet-consistent single-winner election method". In: *Social Choice and Welfare* 36 (2011), pp. 267–303. URL: https://arxiv.org/abs/1804.02973

Wesley H. Holliday and Eric Pacuit. "Split Cycle: a new Condorcet-consistent voting method independent of clones and immune to spoilers". In: *Public Choice* 197.1 (2023), pp. 1–62. DOI: 10.1007/s11127-023-01042-3

Independence of Clones

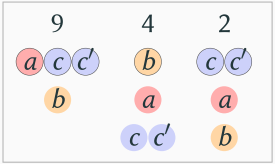


Figure 4: Split-IRV fails independence of clones.

Theorem

Approval-IRV is independent of clones.

Argument similar to linear-order version.

We give a rigorous proof by induction; also shows that linear-order IRV satisfies independence of clones.

Majority Condition

47%	4%	25%	24%
a b	a	c	d
c	b	b	b
d	c	d	c
	d	a	a

Figure 5: A problem with electing majority alternatives.

Linear-order IRV satisfies the *majority criterion*: if a majority of voters places a in top position, then a wins.

How to generalize to weak orders? Maybe "if some candidate is ranked top by a majority, then such a candidate should win"?

In the figure, this implies a is the winner.
But 49% say $b \succ a$ and only 4% say $a \succ b$.

Bad axiom! Need a different generalization.

Approval-IRV: b (also Condorcet winner)
Split-IRV: a

Respect for Cohesive Majorities

9	5	5	8	10
a b c	a b	a c	b c d	d
d	d	d	a	a b c
	c	b		

Figure 6: Split-IRV violates respect for cohesive majorities because it eliminates a, then b and c, and elects d.

Theorem

Approval-IRV respects cohesive majorities.

If a majority of voters rank c on top ("cohesive") then the winner must be ranked top by at least one member of that majority.

Let $\text{top}_i = \{c \in C : c \succcurlyeq_i d \text{ for all } d \in C\}$.

Definition

A voting rule f *respects cohesive majorities* if for all profiles P and all subsets of voters $S \subseteq N$ such that $|S| > \frac{n}{2}$ and $\bigcap_{i \in S} \text{top}_i \neq \emptyset$, we have $f(P) \subseteq \bigcup_{i \in S} \text{top}_i$.

Characterization within Elimination Scoring Rules

Theorem

Approval-IRV is the only elimination scoring rule satisfying independence of clones and respect for cohesive majorities.

The axioms are independent.

An *elimination scoring rule* sequentially eliminates the lowest-scoring candidate, where the scores are positional scores (weakly decreasing) that may be different for each *order type* τ (specifying the sizes of the indifference classes).

Examples: different versions of Borda scoring
Approval: $\tau \mapsto (1, 0, \ldots, 0)$
Split: $\tau \mapsto (1/\tau_1, 0, \ldots, 0)$.

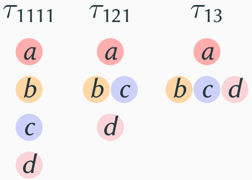


Figure 7: Examples of weak orders with different order types.

John H. Smith. "Aggregation of preferences with variable electorate". In: *Econometrica: Journal of the Econometric Society* 41.6 (1973), pp. 1027–1041. DOI: 10.2307/1914033

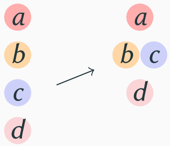


Figure 8: Indifference monotonicity: if c is the winner and a voter makes this change, then c stays winning.

A c -*hover* is the following type of transformation:

$$\begin{aligned} & C_1 \succ \cdots \succ C_j && \succ \{c\} \succ C_{j+2} \succ \cdots \succ C_k \\ \longmapsto \quad & C_1 \succ \cdots \succ C_j \cup \{c\} && \succ C_{j+2} \succ \cdots \succ C_k \end{aligned}$$

Note: c must initially lie in a singleton indifference class.

Definition

A voting rule f is *indifference monotonic* if for every profile P and every $c \in f(P)$, whenever $\hat{P}$ is obtained from P by applying c -hovers to some votes in P, we have $c \in f(\hat{P})$.

Characterization II within Elimination Scoring Rules

Theorem

Approval-IRV is the only elimination scoring rule that agrees with IRV on profiles of linear orders and that is indifference monotonic.

The axioms are independent.

Reasons for using Weak Orders

- *Less effort,* especially true for preferences like $a \succ b \succ \{c, d, e, f\} \succ g$.
- *More expressive,* when voters have true indifferences. (Australia forces no indifferences.)
- *More expressive,* when there are many candidates but at most 5 ranks on the ballot.
- *Fewer invalid ballots.*
- *Better alignment with candidate campaigns,* which typically only ask to be ranked #1, for example in NYC.

> Lindsey Cormack. "More choices, more problems? Ranked choice voting errors in New York City". In: *American Politics Research* (2023). URL: https://dominik-peters.de/archive/cormack2023.pdf

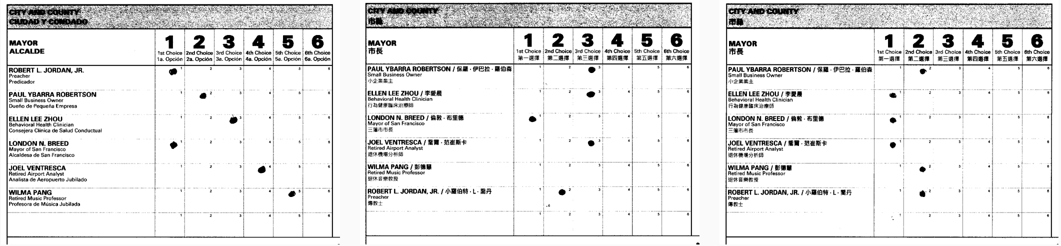
- *Reduce need for some types of strategic voting.*

> Alex Small. "Geometric construction of voting methods that protect voters' first choices". In: *arXiv:1008.4331* (2010). URL: https://arxiv.org/abs/1008.4331

> James Green-Armytage. "Strategic voting and nomination". In: *Social Choice and Welfare* 42 (2014), pp. 111–138. DOI: 10.1007/s00355-013-0725-3. URL: https://link.springer.com/article/10.1007/s00355-013-0725-3

- *A compromise between Ranked Choice Voting and Approval Voting.*

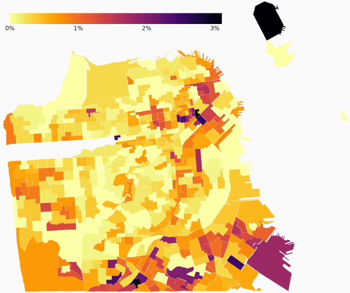
SF Ballots



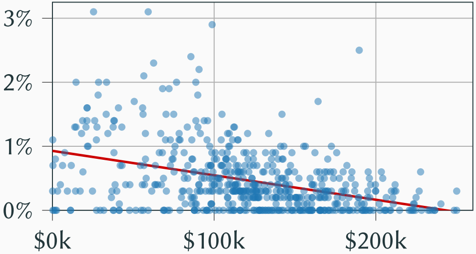
(a) Two top choices **(b)** Vetoing a candidate **(c)** An approval vote

Figure 9: Examples of ballots that can be interpreted as weak orders (2019 mayoral election in San Francisco).

SF: Locations of weak order ballots



(a) Map of San Francisco election precincts, colored by the fraction of votes that could be interpreted as a weak order with indifferences.



(b) Among election precincts, median household income (horizontal axis) is negatively correlated with percent of ballots showing a weak order (vertical axis; $r = -0.4$, $p < 0.001$).

Figure 10: Ballot data from the 2019 mayoral election in San Francisco.

Experiments

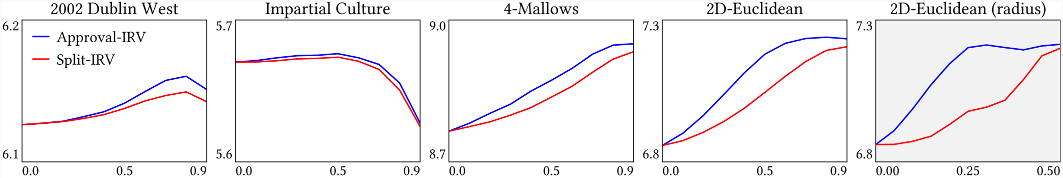


Figure 11: Average Borda score of the winner (normalized by dividing by n) for various datasets.

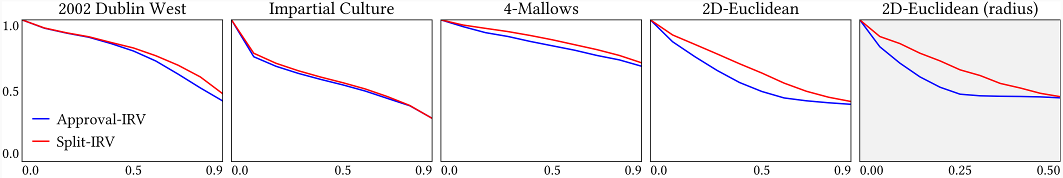


Figure 12: Frequency of agreement between the rule and linear-order IRV for various datasets.

x-axis: few indifferences $\longrightarrow$ *many indifferences*

Experiments

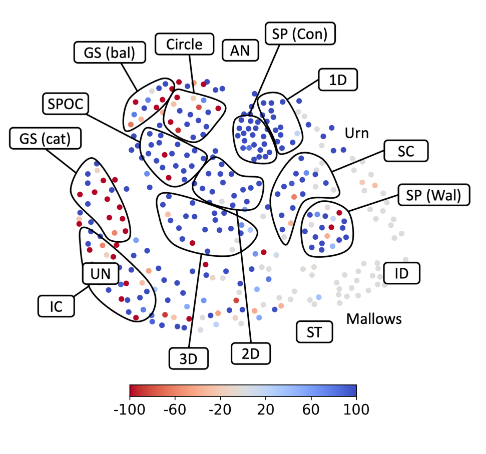


Figure 13: Map of elections, showing the difference in Borda score between the Approval-IRV and Split-IRV winner in the coin-flip model (blue: approval better than split).

Niclas Boehmer et al. “Understanding Distance Measures Among Elections”. In: *Proceedings of the 31st International Joint Conference on Artificial Intelligence (IJCAI)*. 2022, pp. 102–108. DOI: 10.24963/ijcai.2022/15

Multiwinner Version: STV

We can define a weak-order version of the multi-winner rule STV, giving Approval-STV. STV gives proportional representation, which has been formalized via the Proportionality for Solid Coalition property. It has been generalized to weak orders.

> Haris Aziz and Barton E. Lee. "The expanding approvals rule: improving proportional representation and monotonicity". In: *Social Choice and Welfare* 54 (2020), pp. 1–45. DOI: 10.1007/s00355-019-01208-3. URL: https://www.cse.unsw.edu.au/~haziz/prsolution.pdf

Theorem

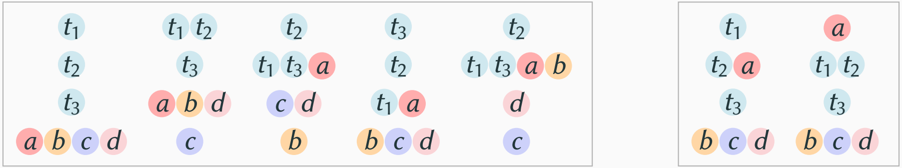
Approval-STV satisfies generalized PSC for weak orders.

Proof of some independent interest also for the linear-order variant.

However, Approval-STV does not satisfy the stronger axiom of rank-PJR.

> Markus Brill and Jannik Peters. "Robust and verifiable proportionality axioms for multiwinner voting". In: *Proceedings of the 24th ACM Conference on Economics and Computation (EC)*. 2023, p. 301. URL: https://arxiv.org/abs/2302.01989

Multiwinner Version: Generalized PSC



voters that are T-supporting — not T-supporting

Figure 14: For $T = \{t_1, t_2, t_3\}$, the 5 voters in the left box are T-supporting, since T is ranked weakly higher than alternatives not in T. The closure of T with respect to the 5 voters is $\{t_1, t_2, t_3, a, b\}$, since some rank a and/or b on the same level as an alternative from T.

Let $q = n/(k+1)$ be the *quota*.

- If $> q$ voters submit ballots from the left-hand box, then generalized PSC requires that at least one t_i or a or b wins.
- For $> 2q$ such voters, at least two of them win.
- For $> 3q$ such voters, at least three of them win.