Hedonic Games with Dichotomous Preferences

Dominik Peters

Department of Computer Science, University of Oxford, UK

- A coalition is an alliance between a group of individuals, formed in order to achieve a common goal.
- Question: How do such coalitions form if agents are selfish?
- Hedonic Games are a widely studied model for coalition formation
- Agents only care about the members of their own coalition (see **box**)
- Previous work on hedonic games is mostly negative: often, stable outcomes don't exist, and finding them is computationally very hard. *Can we do better using dichotomous preferences?*

Stable Outcomes Guaranteed ✓

- **Every dichotomous hedonic game** admits a simultaneously **core-stable** and **individually stable** outcome.
- Outcome is resistant to deviations by both groups and single players.
- Can be found efficiently in many cases.
- An individually stable outcome can always be found efficiently.
- Algorithm: Repeatedly find and assign a coalition that is approved by *all its members*, until no such coalition exists anymore. Then assign the remaining players into one 'loser coalition'.

Boolean Hedonic Games

- Concise representation using logic.
- Example: I approve coalitions satisfying: Alice ∧ ¬Bob ∧ (Charlie → (Dave ∨ Eve))
- Proposed by Aziz et al. (2014/16).
- Aziz et. al. characterise solution concepts using logical formulas that can be fed into a SAT solver
- We provide complexity results (NP-completeness and Σ_2-completeness) that show that these logic-characterisations are optimal.

Few Approved Coalitions

- To chart out the islands of tractability, we consider the case where agents approve only very few coalitions.
- In some contexts, this may also be sensible to assume in practice.
- A k-list consists of at most k approved coalitions.
- We show that welfare-maximisation and finding strict-core-stable partitions is hard even for 1-lists. The former is also inapproximable.
- For 2-lists, 2SAT can find Nash-stable or Pareto-optimal partitions.
- For 3-lists and 4-lists, most problems are already hard.

Anonymous Games

- Here, agents' preferences depend only on coalitions sizes, i.e., the cardinality $|S|$ of their coalition.
- Known to induce hard problems in general case (Ballester 2004).
- We show: NP-hard even for dichotomous anonymous preferences!
- Related to "group activity selection problem" (Darmann et al. 2012).

Hedonic Games: *Definitions*

Finite set N of agents, each agent $i \in N$ having *dichotomous* preferences $\succcurlyeq_i$ over all *groups* $S \subseteq N$.

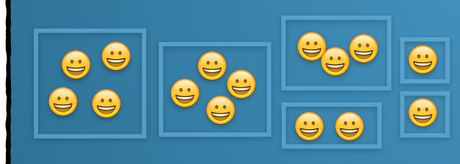
coalitions are split into two 'preference levels' (= indifference classes): approved coalitions and non-approved coalitions

$\{1,2\} \sim_1 \{1,2,3\} \succ_1 \{1\} \sim_1 \{1,3\}$

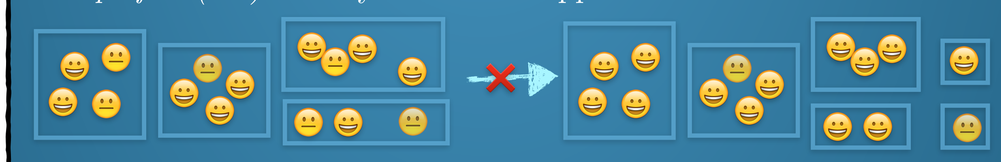
approved coalitions

goal - aligned with preferences: stable & high welfare

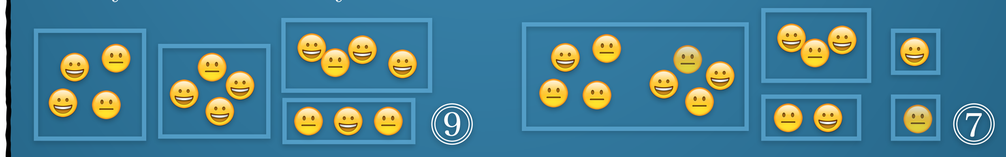
Outcome: a *partition* π of agent set N.



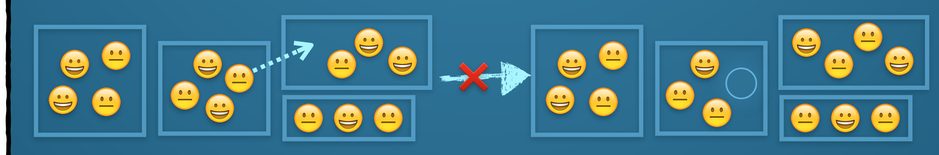
π is *perfect (PF)* if everyone is in an approved coalition



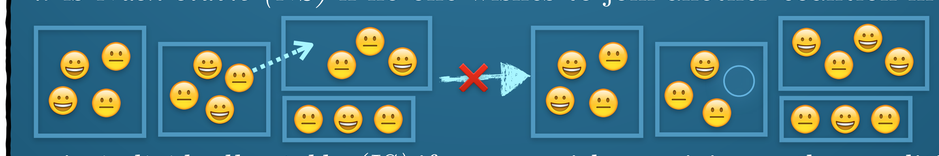
π is *Pareto optimal (PO)* if no π' is strictly better for someone and weakly better for everyone



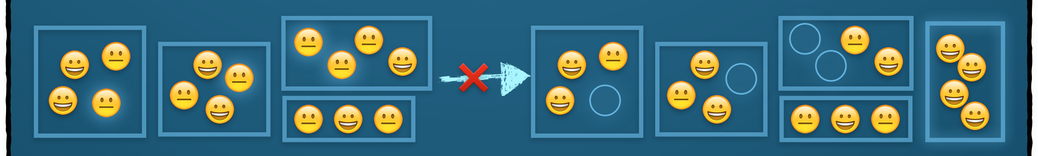
π has *social welfare (SW)* k^* if k^* agents are in approved coalitions



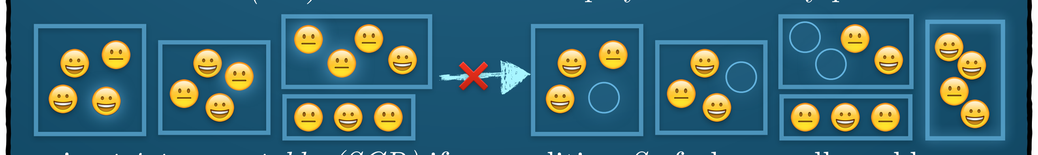
π is *Nash stable (NS)* if no-one wishes to join another coalition in π



π is *individually stable (IS)* if no-one wishes to join another coalition in π all of whose members welcome the deviator



π is *core-stable (CR)* if no coalition S of players all strictly prefer S to π



π is *strict-core-stable (SCR)* if no coalition S of players all weakly prefer S to π, with at least one member of S having a strict preference

Intervals

- Structural restriction: Suppose agents are ordered on a line
- They only approve coalitions that form an interval

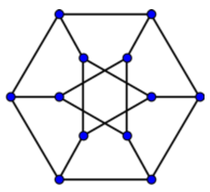
- Dichotomous "single-peakedness" for hedonic games
- Dynamic Programming can produce partitions maximising social welfare, and Pareto-optimal outcomes

Roommates

- Stable Roommates problem: Only allow coalition sizes 1 and 2
- Results from literature imply: most problems (e.g., welfare-maximisation, core-existence, Pareto-optimality) become easy when preferences are dichotomous
- However, finding a Nash stable outcome remains NP-complete

Majority Games

- Preference restriction inspired from social choice / voting.
- Suppose agents sit on the vertices of a graph: they form a social network
- Agents want to be influential in their coalition: approve those coalitions in which they are adjacent to majority of players



- Intuition: Players are happy when their opinion would prevail in majority elections within their coalition.
- Analysed using Hajnal-Szemerédi thm from extremal graph theory and graph packing algorithms (Hell & Kirkpatrick).
- Guaranteed to have a simultaneously Nash-stable and core-stable outcome which can be found efficiently.
- Can also efficiently find a perfect or strict-core-stable outcome if one exists.

Conclusions

- Dichotomous preferences can avoid the complexity trap in hedonic games, at least when requiring additional structure
- Future work: Find other promising structural restrictions, perhaps using a graphical approach. Evaluate Boolean Hedonic Games using state-of-the-art SAT solvers.

	SW	PF	PO	NS	IS	CR	SCR
Boolean	NP-c.	NP-c.	NP-h.	NP-c.	P	FNP-h.	Σ_2^p-c.
1-lists	NP-c.	P	P	P	P	P	NP-c.
2-lists	NP-c.	P	P	P	P	P	NP-c.
3-lists	NP-c.	NP-c.	NP-h.	?	P	P	NP-c.
4-lists	NP-c.	NP-c.	NP-h.	NP-c.	P	P	NP-c.
Anonymous	NP-c.	NP-c.	NP-h.	NP-c.	P	P	NP-c.
Intervals	P	P	P	?	P	P	?
Roommates	P	P	P	NP-c.	P	P	P
Majority	?	P	?	P	P	P	P

Presented at AAAI 2016, Phoenix, Arizona