Proportionality and Strategyproofness in Multiwinner Elections

Dominik Peters
University of Oxford

Multiwinner Elections

- Given approval profile, select a committee of exactly *k* candidates.
- Find good committee *rule:* $f : (2^A)^n \to \mathscr{C}_k$
- One option (AV): choose *k* candidates with highest approval score
  — strategyproof!
- e.g. (*ab, ab, ab, de, df*) -> AV committee *ab*; proportional *ad*

Proportional Approval Voting (PAV)

- Select the k-committee W maximising

$$\sum_{i \in N} \left(1 + \frac{1}{2} + \cdots + \frac{1}{|P(i) \cap W|} \right)$$

- Diminishing marginal utility
- Proportional: 30% group gets ~30% of seats
- Vulnerable to free-riding
  Profile (*abc*, *abc*, *abc*, *abd*, *abd*) -> *abc*
  (*abc*, *abc*, *abc*, *abd*, *d*) -> *abd*

Is there a rule which is both proportional and strategyproof?

(No.)

Is there a rule which is both proportional and strategyproof?

Theorem. Suppose $k \geq 3$ and n is a multiple of k. No resolute rule satisfies proportionality and strategyproofness.

Strategyproofness

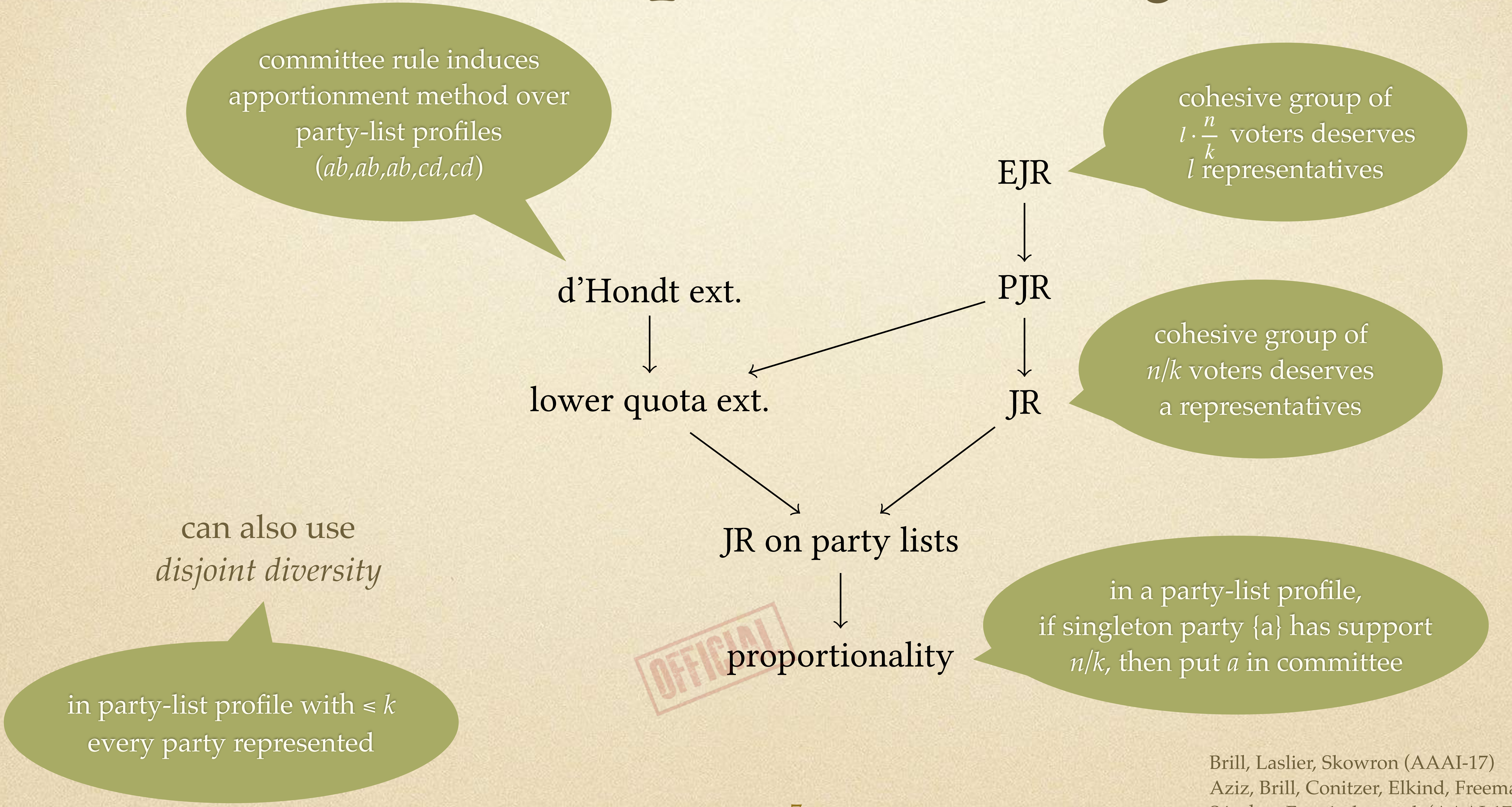
Cardinality-Strategyproofness If P' is an i-variant of P, then we do not have $|f(P') \cap P(i)| > |f(P) \cap P(i)|$.

Hamming-Strategyproofness If P' is an i-variant of P, then we do not have $\mathcal{H}(f(P'), P(i)) < \mathcal{H}(f(P), P(i))$.

Superset-Strategyproofness If P' is an i-variant of P, then we do not have $f(P') \cap P(i) \supsetneq f(P) \cap P(i)$.

Strategyproofness If P' is an i-variant of P with $P'(i) \subset P(i)$, then we do not have $f(P') \cap P(i) \supsetneq f(P) \cap P(i)$.

Proportionality



Brill, Laslier, Skowron (AAAI-17)
Aziz, Brill, Conitzer, Elkind, Freeman, Walsh (SCW-17)
Sánchez-Fernández et al. (AAAI-17)

SAT Solving

ALGORITHM 1: Encode Problem for SAT Solving

Input: Set C of candidates, set N of voters, committee size k.
Question: Does a proportional and strategyproof committee rule exist?

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for each profile P ∈ 𝓑^N do
    if P is a party-list profile then
        allowed[P] ← {C ∈ C_k : C provides JR to singleton parties}
    else
        allowed[P] ← C_k
    for each committee C ∈ allowed[P] do
        introduce propositional variable x_{P,C}
for each profile P ∈ 𝓑^N do
    add clause ⋁_{C∈allowed[P]} x_{P,C}
    add clauses ⋀_{C≠C'∈allowed[P]} (¬x_{P,C} ∨ ¬x_{P,C'})
    for each voter i ∈ N do
        for each i-variant P' of P with P'(i) ⊆ P(i) do
            for each C ∈ allowed[P] and C' ∈ allowed[P'] do
                if C' ∩ P(i) ⊋ C ∩ P(i) then
                    add clause (¬x_{P,C} ∨ ¬x_{P',C'})
pass formula to SAT solver
return whether formula is satisfiable
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Applications to:

- Tournament solutions
- Voting
- Randomised voting
- Fair budgeting
- House allocation

SAT -> good rule
UNSAT -> impossibility

Minimal Unsatisfiable Set yields human-readable proof

Proof Strategy

- Using SAT + MUS, prove base case $k = 3$, $n = 3$, $m = 4$.
  - Consider profile $P = (ab, c, d)$; by proportionality wlog $f(P) = acd$.
  - Use strategyproofness to deduce value of f at 13 other profiles, obtaining contradiction $f(P) = bcd$.
- Prove induction steps:
  - By copying the profile, holds for $q \cdot k$ voters
  - By adding never-approved candidates, holds for $m \geq 3$
  - By adding a k+1st ‘party’, can move from k to $k + 1$

$k = 2$?

other n?

what can we say without?

Theorem. Suppose $k \geq 3$ and n is a multiple of k. No resolute rule satisfies proportionality and strategyproofness.

Small Committees

- Result does not hold for $k = 2$.
- For $k = 2$ and n odd, AV satisfies JR!
  - because cohesive groups are strict majorities, which AV favours
- But does hold when allowing all manipulations (not just subsets).

Other Electorate Sizes

- Is the assumption that n is a multiple of k necessary?
  - SAT solver suggests yes, at least for small parameters
  - Open question
- Result holds for *all* n when using Droop quota: every group of $> n/(k+1)$ needs to be represented.

Resoluteness

- Proof exploits "awkward resoluteness"
  $P = (ab, c, d)$; f needs to make an arbitrary choice
- How to relax resoluteness?
- Result holds for randomised rules:
  - Proportionality: all committees in support must be proportional
  - Strategyproofness: Can't get more approved candidates in expectation

Open Questions

- Does result hold for all n?
- What about single-peaked preferences?

  intuitively, choosing e.g. percentiles is proportional
- Proportionality and (strong) monotonicity
- EJR and committee monotonicity
- Characterise AV with reinforcement + s.p. (cf. Lackner & Skowron IJCAI-18)