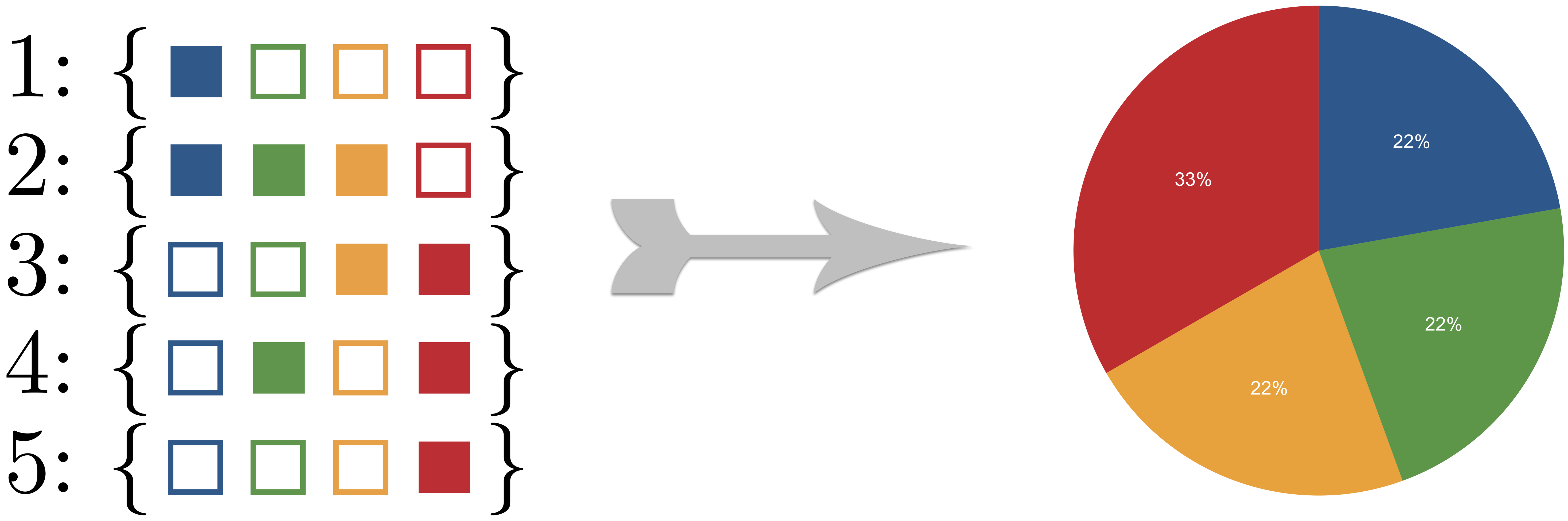
Price of Fairness in Budget Division

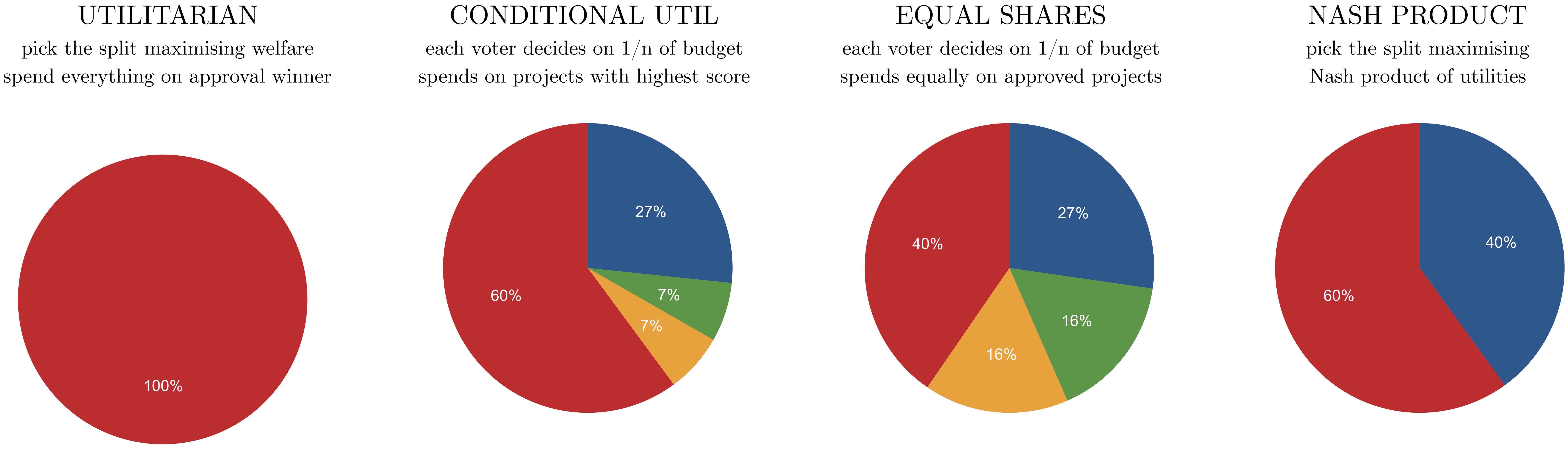
Marcin Michorzewski
University of Warsaw

Dominik Peters
Carnegie Mellon University

Piotr Skowron
University of Warsaw

We need to divide a budget among projects. Voters want money to be spent on approved projects.





The model:

- The set of voters: $N = \{1, \dots, n\}$
- The set of objects: $O = \{o_1, \dots, o_m\}$
- Each voter i approves a subset A_i of objects

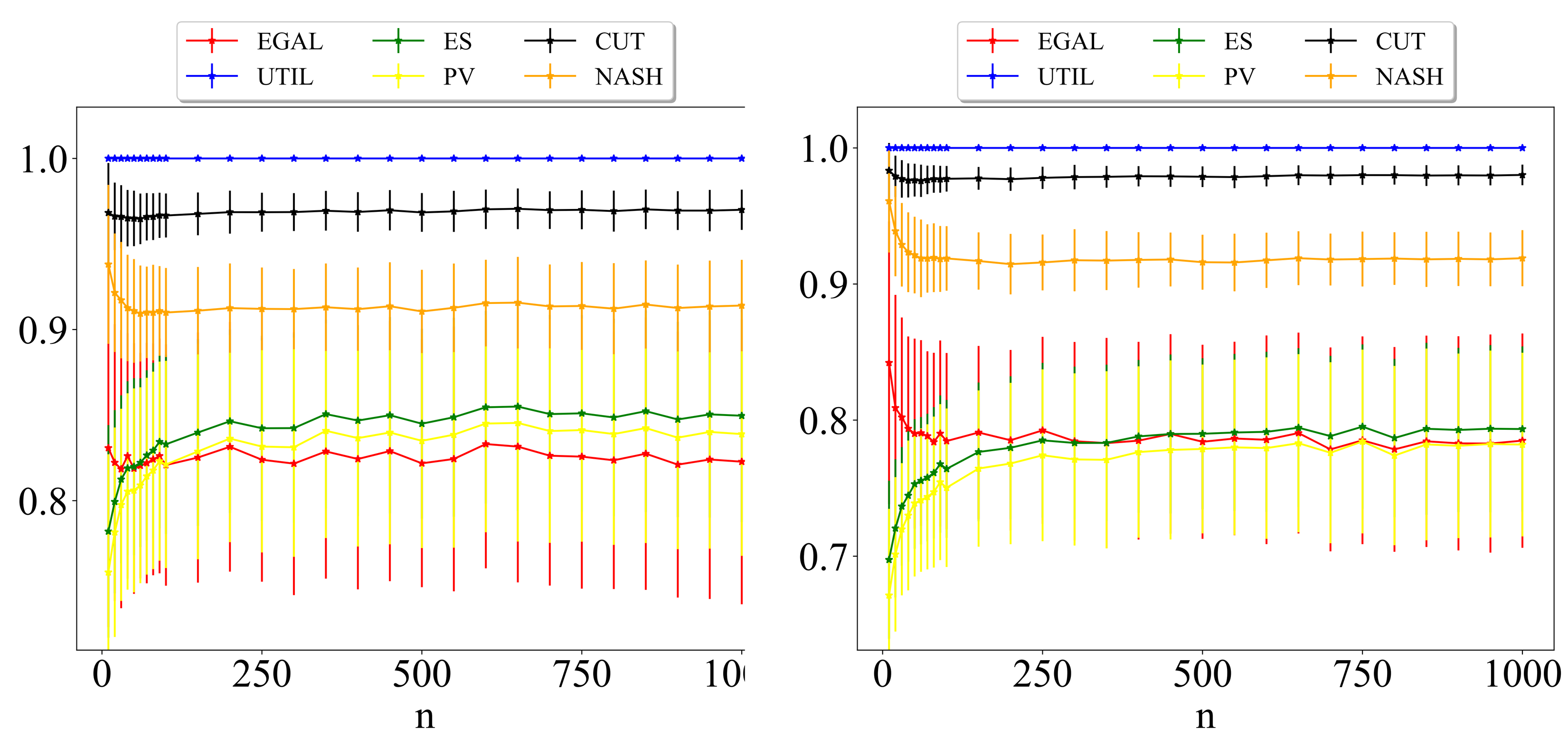
Given the lottery $\mathbf{x} = (x_1, \dots, x_m)$:

- The utility of voter i is $u_i(\mathbf{x}) = \sum_{o_j \in A_i} x_j$
- The total utility of voters is $u(\mathbf{x}) = \sum_{i \in N} u_i(\mathbf{x})$

	UTIL	CUT	NASH	EGAL	PV	ES
IFS		+	+	+		+
GFS		+	+			+
impl.		+	+			+
AFS			+			
core			+			

Table 1: Voting rules, and the fairness axioms they satisfy

The utilitarian rule picks the lottery $\mathbf{x}$ that maximizes the total utility of the voters, but this rule is not fair.
Q: What is the (worst-case) cost (in terms of utilitarian social welfare) of imposing the fairness axioms?
A: For (1) individual fair share, (2) average fair share, and (3) the core, it is $\Theta\left(\frac{1}{\sqrt{m}}\right)$.



There are rules that implement a smooth tradeoff between utilitarian efficiency and fairness.
The paper describes these rules, and characterises the tradeoff.

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